100 學年四技二專第二次聯合模擬考試
共同考科　數學(C)卷　詳解

數學(C)卷 100-2-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	B	D	D	C	C	A	D	D	C	A	C	A	D	A	D	C	C	B	B	B	C	B	A	A

1. 設 C 點坐標 (x,y)
$5\overline{AC}=7\overline{BC}\Rightarrow\overline{AC}：\overline{BC}=7：5\Rightarrow\overline{AB}：\overline{BC}=2：5$
則 $-1=\dfrac{2(x)+5(-3)}{2+5}\Rightarrow x=4$
$3=\dfrac{2(y)+5(5)}{2+5}\Rightarrow y=-2$

2. $f(x)=4-\cos x-(1-\cos^2 x)=\cos^2 x-\cos x+3$
$=(\cos x-\dfrac{1}{2})^2+\dfrac{11}{4}$，$\because -1\le\cos x\le 1$
當 $\cos x=-1$ 時，$f(x)$ 有最大值 $M=(-1-\dfrac{1}{2})^2+\dfrac{11}{4}=5$
當 $\cos x=\dfrac{1}{2}$ 時，$f(x)$ 有最小值 $m=(\dfrac{1}{2}-\dfrac{1}{2})^2+\dfrac{11}{4}=\dfrac{11}{4}$

3. $2\sin\theta=-3\cot\theta\Rightarrow 2\sin\theta=-3\dfrac{\cos\theta}{\sin\theta}$
$\Rightarrow 2\sin^2\theta+3\cos\theta=0$
$2(1-\cos^2\theta)+3\cos\theta=0\Rightarrow(2\cos\theta+1)(\cos\theta-2)=0$
$\Rightarrow\begin{cases}\cos\theta=-\dfrac{1}{2}\\ \cos\theta=-2\,(\text{不合})\end{cases}$，當 $\cos\theta=-\dfrac{1}{2}$ 時，$\theta=\dfrac{2\pi}{3}$

4. $\begin{cases}\text{兩根和}\sin\theta+\cos\theta=-\dfrac{k}{5}>0\ (\because\theta\text{爲銳角})\\ \text{兩根積}\sin\theta\cdot\cos\theta=\dfrac{2}{5}\end{cases}$
$\because(\sin\theta+\cos\theta)^2=\sin^2\theta+2\sin\theta\cdot\cos\theta+\cos^2\theta$
$=1+2\sin\theta\cdot\cos\theta$
$(-\dfrac{k}{5})^2=1+2\times\dfrac{2}{5}\Rightarrow\dfrac{k^2}{25}=\dfrac{9}{5}$
$\Rightarrow k=\pm3\sqrt{5}$ (正不合，$\because k<0$)

5. $\tan\theta<0$ ，$\cos\theta<0$ ，則 $\theta\in\text{II}$
知 $x=-4$，$y=3$，$r=5$，所求 $=5\times(\dfrac{3}{5})+4\cdot(-\dfrac{5}{4})=-2$

6. $\cos B=\dfrac{c^2+a^2-b^2}{2ca}\Rightarrow\dfrac{(3)^2+(7)^2-b^2}{2\times(3)\times(7)}=\dfrac{11}{14}$
$b^2=25$，$\therefore b=5$
$\cos A=\dfrac{b^2+c^2-a^2}{2bc}=\dfrac{(5)^2+(3)^2-(7)^2}{2\times(5)\times(3)}=-\dfrac{1}{2}$
$\therefore\angle A=120^\circ$

7. $|2\vec{a}-\vec{b}|=\sqrt{37}\Rightarrow 4|\vec{a}|^2-4\vec{a}\cdot\vec{b}+|\vec{b}|^2=37$
$\Rightarrow\vec{a}\cdot\vec{b}=-3\Rightarrow\cos\theta=\dfrac{-3}{2\times3}\Rightarrow\theta=120^\circ$

8. $\left|\dfrac{1}{2}\begin{vmatrix}7&1\\3&k\end{vmatrix}\right|=\dfrac{25}{2}\Rightarrow|7k-3|=25\Rightarrow k=4$ 或 $-\dfrac{22}{7}$

9. 令 $\overrightarrow{AB}=\vec{t}$，$\vec{t}$ 與相反方向之單位向量爲 $-\dfrac{\vec{t}}{|\vec{t}|}$
$\therefore\vec{v}=2(-\dfrac{\vec{t}}{|\vec{t}|})=2(-\dfrac{(1,-1)}{\sqrt{2}})=(-\sqrt{2},\sqrt{2})$

10. 有 $-1+i$ 之根，另一根爲 $-1-i$
(實係數方程式虛根成雙)
由根與係數關係知：
$\begin{cases}(-1+i)+(-1-i)=-a\\(-1+i)(-1-i)=b\end{cases}\Rightarrow\begin{cases}a=2\\b=2\end{cases}$
$\therefore a-b=2-2=0$

11. 設 $f(x)=a(x-2)(x+3)+(2x+3)$
$f(0)=9\Rightarrow a(-2)(3)+(0+3)=9\Rightarrow a=-1$
$\therefore f(x)=-(x-2)(x+3)+(2x+3)$
$\Rightarrow f(-1)=-(-1-2)(-1+3)+(2(-1)+3)=7$

12. 令 $x+\dfrac{1}{x}=t\Rightarrow x^2+\dfrac{1}{x^2}=t^2-2$
$x^3+\dfrac{1}{x^3}=(x+\dfrac{1}{x})(x^2-x\cdot\dfrac{1}{x}+\dfrac{1}{x^2})=18$
$\because\Rightarrow t(t^2-3)=18\Rightarrow t^3-3t-18=0$
$\Rightarrow(t-3)(t^2+3t+6)=0\Rightarrow t=3(t^2+3t+6=0$ 無實根)

13. $\begin{cases}\alpha+\beta=-3\\\alpha\beta=-2\end{cases}\Rightarrow\alpha^2+3\alpha-2=0\Rightarrow\alpha^2-2=-3\alpha$
同理 $\beta^2-2=-3\beta$
$\begin{cases}\text{新方程式兩根和}\\=(\alpha^2-2)+(\beta^2-2)=-3(\alpha+\beta)=9\\\text{新方程式兩根積}\\=(\alpha^2-2)(\beta^2-2)=(-3\alpha)(-3\beta)=9\times(-2)=-18\end{cases}$
故所求方程式爲 $x^2-9x-18=0$

14. 原式 $=\dfrac{(a^x)^2-(a^{-x})^2}{(a^x)+(a^{-x})}=\dfrac{(a^x+a^{-x})(a^x-a^{-x})}{(a^x+a^{-x})}$
$=(a^x-a^{-x})=3-\dfrac{1}{3}=\dfrac{8}{3}$

15. $a=3^{\frac{1}{2}}$ ，$b=3^{\frac{2}{3}}$ ，$c=2^{\frac{3}{4}}$
底數 $3>1$，$\log_3 x$ 爲遞增函數

$\because \frac{2}{3} > \frac{1}{2}$，$\therefore 3^{\frac{2}{3}} > 3^{\frac{1}{2}}$，$\Rightarrow b > a \cdots\cdots(1)$

又 $a = 3^{\frac{1}{2}} = (3^2)^{\frac{1}{4}} = 9^{\frac{1}{4}}$，$c = 2^{\frac{3}{4}} = (2^3)^{\frac{1}{4}} = 8^{\frac{1}{4}}$

$\Rightarrow a > c \cdots\cdots(2)$

由(1)、(2)得 $c < a < b$

16. $\log(x+1) + \log(x-2) = \log 10$

$\Rightarrow \log(x+1)(x-2) = \log 10$

$\Rightarrow x^2 - x - 2 = 10 \Rightarrow x^2 - x - 12 = 0$

$\Rightarrow (x-4)(x+3) = 0 \Rightarrow x = 4$ 或 $x = -3$

∵眞數 > 0，$\therefore x = 4$ (合)，$x = -3$ (不合)

∴所有解之和爲 4

17. $\because 2.1 < \log A < 2.2$

$\log A^3 = 3\log A \Rightarrow 6.3 < 3\log A < 6.6$

$\log A^3$ 之首數爲 6，∴A 爲 $6+1=7$ 位數

18. $S_{15} = \frac{15(a_1 + a_{15})}{2} \Rightarrow 660 = \frac{15(a_1 + 16)}{2} \Rightarrow a_1 = 72$

$\Rightarrow d = \frac{a_{15} - a_1}{15-1} = -4$

$a_n = a_1 + (n-1)d = 72 - 4(n-1) < 0$

$\Rightarrow 4n > 76 \Rightarrow n > \frac{76}{4} = 19$，$n$ 取 20

19. 公比 $r = \frac{1}{3} \div \frac{1}{9} = 3 \Rightarrow 729 = \frac{1}{9} \cdot 3^{n-1}$

$\Rightarrow 3^{n-3} = 3^6 \Rightarrow n = 9$

20. 設 $D(x,y)$，有三種可能性

① 當 $A+B=C+D$ 時，

$\begin{cases} 8+(-2) = -4+x \\ 4+6 = 2+y \end{cases} \Rightarrow \begin{cases} x=10 \\ y=8 \end{cases} \Rightarrow D(10,8)$

② 當 $A+C=B+D$ 時，

$\begin{cases} 8+(-4) = -2+x \\ 4+2 = 6+y \end{cases} \Rightarrow \begin{cases} x=6 \\ y=0 \end{cases} \Rightarrow D(6,0)$

③ 當 $B+C=A+D$ 時，

$\begin{cases} (-2)+(-4) = 8+x \\ 6+2 = 4+y \end{cases} \Rightarrow \begin{cases} x=-14 \\ y=4 \end{cases} \Rightarrow D(-14,4)$

21. $\tan 2\theta = \frac{3}{4} \Rightarrow \sin 2\theta = \frac{3}{5}$

所求 $= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta = 1 + \sin 2\theta$

$= 1 + \frac{3}{5} = \frac{8}{5}$

22. 令 $t = \sin x + \cos x \Rightarrow -\sqrt{2} \le t \le \sqrt{2}$

$t^2 = (\sin x + \cos x)^2 \Rightarrow \sin 2x = 2\sin x\cos x = t^2 - 1$

$f(x) = \sin 2x + 2\sin x + 2\cos x = t^2 + 2t - 1 = (t+1)^2 - 2$

當 $t = \sqrt{2}$ 時，$f(x)$ 有最大値 $2\sqrt{2}+1$

23. 原式 $= \frac{(3+2i)(1+i)}{(1-i)(1+i)} = \frac{3+2i+3i-2}{1^2+1^2} = \frac{1}{2} + \frac{5}{2}i$

$\therefore a = \frac{1}{2}$，$b = \frac{5}{2}$，所求 $a - b = \frac{1}{2} - \frac{5}{2} = -2$

24. 原式 $= \left| \frac{(3-4i)^2(1-i)^3}{(1+i)(8+6i)} \right|$

$= \frac{(\sqrt{3^2+(-4)^2})^2 \cdot (\sqrt{1^2+(-1)^2})^3}{\sqrt{1^2+1^2} \cdot \sqrt{8^2+6^2}} = \frac{5^2 \cdot (\sqrt{2})^3}{\sqrt{2} \cdot 10} = 5$

25. $1 - i = \sqrt{1^2+(-1)^2}[\frac{1}{\sqrt{2}} + (-\frac{1}{\sqrt{2}})i]$，$\cos\theta = \frac{1}{\sqrt{2}}$

$\sin\theta = -\frac{1}{\sqrt{2}} \Rightarrow$ 取 $\theta = -\frac{\pi}{4}$

$1 - i = \sqrt{2}[\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})]$