102 學年四技二專第五次聯合模擬考試
共同科目　數學(C)卷　詳解

數學(C)卷 102-5-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	C	B	D	C	C	B	A	D	A	D	A	A	C	B	A	D	C	D	B	A	C	B	D	B

1. $\begin{cases} y=2^{3x-1} \\ y=4^{x-1} \end{cases} \Rightarrow 2^{3x-1}=4^{x-1} \Rightarrow x=-1$ 、 $y=\dfrac{1}{16}$

$\Rightarrow (a\,,b)=(-1\,,\dfrac{1}{16}) \Rightarrow a+b=\dfrac{-15}{16}$

2. $f\,'(x)=\dfrac{1}{3}\times(x^2-1)^{-\frac{2}{3}}\times(2x)$ ， $f\,'(3)=\dfrac{1}{2}$

切線的斜率爲 $\dfrac{1}{2}$ ，且通過 $(3\,,2)$

切線方程式爲 $y-2=\dfrac{1}{2}(x-3) \Rightarrow x-2y+1=0$

3. $\dfrac{80+82+73+85}{4}=80$ ……平時成績之平均值

$80\times0.4+83\times0.2+90\times0.2+72\times0.2=81$ 分

4. 設 $\vec{a}$ 與 $\vec{b}$ 之夾角爲 θ ， $|3\vec{a}-2\vec{b}|^2=3^2$

$\Rightarrow 9|\vec{a}|^2-12\vec{a}\cdot\vec{b}+4|\vec{b}|^2=9 \Rightarrow \vec{a}\cdot\vec{b}=3$

$\cos\theta=\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\dfrac{3}{1\times3}=1 \Rightarrow \theta=0°$

5. 甲、乙、丙爲獨立事件

$P(甲\cap乙\cap丙')+P(甲\cap乙'\cap丙)+P(甲'\cap乙\cap丙)$

$=P(甲)\times P(乙)\times P(丙')+P(甲)\times P(乙')\times P(丙)$

$+P(甲')\times P(乙)\times P(丙)$

$=\dfrac{3}{4}\times\dfrac{2}{3}\times\dfrac{1}{2}+\dfrac{3}{4}\times\dfrac{1}{3}\times\dfrac{1}{2}+\dfrac{1}{4}\times\dfrac{2}{3}\times\dfrac{1}{2}=\dfrac{11}{24}$

6. 原式 $=(3\log_3 2-\log_3 2)\times(\dfrac{3}{2}\log_2 3+\log_2 3)$

$=2\log_3 2\times\dfrac{5}{2}\log_2 3=5$

7. $f\,'(3)=\lim\limits_{x\to3}\dfrac{f(x)-f(3)}{x-3}=\lim\limits_{x\to3}\dfrac{\dfrac{(x-3)(x-4)(x-5)}{x(x-1)(x-2)}-0}{x-3}$

$=\lim\limits_{x\to3}\dfrac{(x-4)(x-5)}{x(x-1)(x-2)}=\dfrac{1}{3}$

8. $16x^2+25y^2-128x-50y-119=0$

$\Rightarrow 16(x^2-8x)+25(y^2-2y)=119$

$\Rightarrow 16(x-4)^2+25(y-1)^2=400$

$\Rightarrow \dfrac{(x-4)^2}{25}+\dfrac{(y-1)^2}{16}=1 \Rightarrow a^2=25 \Rightarrow a=5$

$\therefore \overline{PF}+\overline{PF'}=2a=10$

9. $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta \Rightarrow 1-3x=\sqrt{10}\times\sqrt{1+x^2}\times(-\dfrac{\sqrt{2}}{2})$

$\Rightarrow 1-6x+9x^2=5+5x^2 \Rightarrow 2x^2-3x-2=0$

$\Rightarrow (x-2)(2x+1)=0 \Rightarrow x=2$ 或 $x=-\dfrac{1}{2}$ (代入不合)

10. 設 C 點坐標 $(x\,,0)$

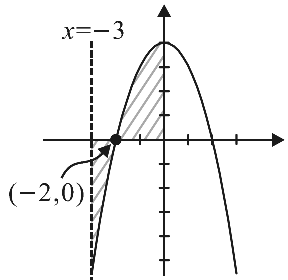
$\overline{AC}=\overline{BC} \Rightarrow \sqrt{(x-5)^2+(0-(-2))^2}=\sqrt{(x-3)^2+(0-4)^2}$

$\Rightarrow x^2-10x+25+4=x^2-6x+9+16 \Rightarrow 4x=4 \Rightarrow x=1$

$\therefore C(1\,,0)$

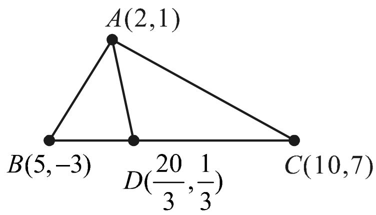
11. $\int_{-3}^{-2}(x^2-4)dx+\int_{-2}^{0}(4-x^2)dx$

$=(\dfrac{1}{3}x^3-4x)\Big|_{-3}^{-2}+(4x-\dfrac{1}{3}x^3)\Big|_{-2}^{0}=\dfrac{7}{3}+\dfrac{16}{3}=\dfrac{23}{3}$



12. $\overline{BD}：\overline{CD}=\overline{AB}：\overline{AC}=5：10=1：2$

D 點坐標 $(\dfrac{1\times10+2\times5}{1+2}\,,\dfrac{1\times7+2\times(-3)}{1+2})=(\dfrac{20}{3}\,,\dfrac{1}{3})$



13. $(kx^2+\dfrac{1}{x})^5$ 的一般項爲 $C_r^5(kx^2)^{5-r}(\dfrac{1}{x})^r=C_r^5k^{5-r}x^{10-3r}$

$\because x$ 項的係數爲 90

令 $10-3r=1 \Rightarrow r=3 \Rightarrow C_3^5k^2=90 \Rightarrow k^2=9$

$\Rightarrow k=\pm3$ (負不合)

14. 原式 $=\dfrac{\cos80°+i\sin80°}{\cos140°+i\sin140°}=\cos(-60°)+i\sin(-60°)$

$=\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}i$

$\Rightarrow a=\dfrac{1}{2}$ 、 $b=-\dfrac{\sqrt{3}}{2} \Rightarrow a^2+b^2=1$

15. $\begin{cases} a_1+a_3+a_5+a_7+a_9=20 \;\cdots\cdots ① \\ a_2+a_4+a_6+a_8+a_{10}=40 \;\cdots\cdots ② \end{cases}$

②－①， $5d=20 \Rightarrow d=4$

16. 利用 $\cos^2 x = 1 - \sin^2 x$

$\therefore$原式 $= -\sin^2 x + \sin x = -(\sin^2 x - \sin x)$

$= -(\sin x - \frac{1}{2})^2 + \frac{1}{4}$

當 $\sin x = \frac{1}{2}$ 時，$f(x)$ 有最大值 $\frac{1}{4}$

17. 圓 C：$(x-2)^2 + (y+3)^2 = 2^2$

圓心 $O(2,-3)$，半徑爲 2

$d(O, L) = \frac{|8+9+3|}{\sqrt{4^2 + (-3)^2}} = 4$

最遠距離 $M = 4 + 2 = 6$

最近距離 $m = 4 - 2 = 2$

$\therefore M + m = 6 + 2 = 8$

18. 等式兩邊同乘 $(x+1)(x-1)^2$

$\Rightarrow x + 3 = a(x-1)^2 + b(x+1)(x-1) + c(x+1)$

$x = 1$ 代入，$4 = 2c$，$\therefore c = 2$

$x = -1$ 代入，$2 = 4a$，$\therefore a = \frac{1}{2}$

比較 x^2 項係數 $\Rightarrow b = -\frac{1}{2}$，$a + b + c = 2$

19. 取出 1 個硬幣的期望值爲

$50 \times \frac{2}{10} + 10 \times \frac{3}{10} + 5 \times \frac{5}{10} = \frac{31}{2}$

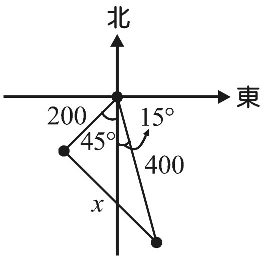
$\therefore$取出 2 個硬幣的期望值爲 $\frac{31}{2} \times 2 = 31$

20. 設此期間颱風移動 x 公里

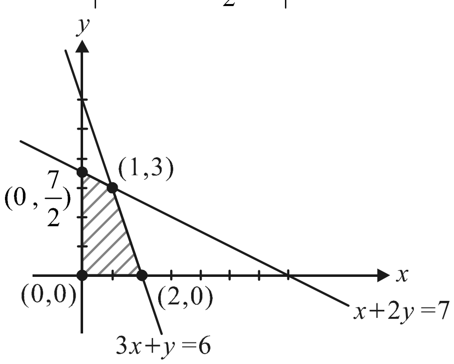
$x^2 = 400^2 + 200^2 - 2 \times 400 \times 200 \times \cos 60° = 120000$

$\Rightarrow x = 200\sqrt{3}$

平均時速 $= \frac{200\sqrt{3}}{20} = 10\sqrt{3}$



21. 面積 $= \frac{1}{2}\begin{vmatrix} 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & \frac{7}{2} & 0 \end{vmatrix} = \frac{19}{4}$



22. $\begin{vmatrix} 2-x & 3 & 4 \\ 2 & 3-x & 4 \\ 2 & 3 & 4-x \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 9-x & 3 & 4 \\ 9-x & 3-x & 4 \\ 9-x & 3 & 4-x \end{vmatrix} = 0$ （第 2 行 $\times 1$、第 3 行 $\times 1$ 加到第 1 行）

$\Rightarrow (9-x)\begin{vmatrix} 1 & 3 & 4 \\ 1 & 3-x & 4 \\ 1 & 3 & 4-x \end{vmatrix} = 0$ （第 1 列 $\times(-1)$ 加到第 2 列、第 3 列）

$\Rightarrow (9-x)\begin{vmatrix} 1 & 3 & 4 \\ 0 & -x & 0 \\ 0 & 0 & -x \end{vmatrix} = 0 \Rightarrow (9-x)x^2 = 0$

$\Rightarrow x = 9$ 或 $x = 0$ (不合)

23. 原式 $= \frac{\sin\theta}{\cos\theta} + \frac{-\tan\theta}{\tan\theta} + \frac{-\sin\theta}{\cos\theta} = -1$

24. 設 $f(x) = x^7 - 10x^6 + 12x^5 - 25x^4 - 21x^3 + 32x^2 - 46x^1 + 109$

原式 $= f(9)$，又 $f(9)$ 爲 $f(x)$ 除以 $x - 9$ 之餘式

1	−10	+12	−25	−21	+32	−46	+109	9
	+9	−9	+27	+18	−27	+45	−9	
1	−1	+3	+2	−3	+5	−1	+100	

$\therefore$所求 $= 100$

25. $[(b+c)+a][(b+c)-a] = 3bc$

$\Rightarrow (b+c)^2 - a^2 = 3bc \Rightarrow b^2 + c^2 - a^2 = bc$

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{bc}{2bc} = \frac{1}{2}$

$\therefore \angle A = 60°$