106 學年度四技二專第二次聯合模擬考試
共同科目　數學(C)卷　詳解

數學(C)卷 106-2-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	D	B	C	D	D	A	C	B	B	A	D	C	C	C	A	B	D	A	B	D	A	C	D

1. $\overline{AB}$ 的中點爲 $P(\frac{a}{2},\frac{b}{2})$
   $\because$ 直線 $2x-y=5$ 爲 $\overline{AB}$ 的中垂線
   $\therefore P$ 在直線 $2x-y=5$ 上
   $\Rightarrow 2\cdot(\frac{a}{2})-\frac{b}{2}=5 \Rightarrow 2a-b=10$ ……①
   又 $\overline{AB}$ 與直線 $2x-y=5$ 垂直
   $\Rightarrow \frac{b-0}{a-0}\cdot 2=-1 \Rightarrow a=-2b$ ……②
   解①②得 $a=4$，$b=-2$，故 $a+b=2$

2. $\overline{AB}=2\overline{BC} \Rightarrow \frac{\overline{AB}}{\overline{BC}}=\frac{2}{1}$
   $\because B$ 在 $\overline{AC}$ 上
   $\therefore$ 由內分點公式得 $B(4,-2)=(\frac{2\cdot m+1\cdot 0}{2+1},\frac{2\cdot n+1\cdot 6}{2+1})$
   $\Rightarrow 2m=12$，$2n+6=-6 \Rightarrow m=6$，$n=-6$
   故 $m-n=12$

3. $\because m_1=\tan\theta_1$，$m_2=\tan\theta_2$，$m_3=\tan\theta_3$，$m_4=\tan\theta_4$
   且 $m_4>m_3>0>m_2>m_1$
   $\Rightarrow \tan\theta_4>\tan\theta_3>0>\tan\theta_2>\tan\theta_1$
   $\Rightarrow \pi>\theta_2>\theta_1>\frac{\pi}{2}>\theta_4>\theta_3>0$

4. $\because \tan\theta<0$ 且 $\sec=\frac{5}{3}>0 \Rightarrow \theta$ 爲第四象限角
   又 $\sec\theta=\frac{5}{3}=\frac{r}{12} \Rightarrow r=20$，$\therefore a=-\sqrt{20^2-12^2}=-16$

5. 原式 $=(\cos^2 25^\circ+\sin^2 25^\circ-\sec^2 35^\circ)\cdot\tan^2 55^\circ$
   $=(1-\sec^2 35^\circ)\cdot\tan^2 55^\circ=(-\tan^2 35^\circ)\cdot\tan^2 55^\circ$
   $=-\tan^2 35^\circ\cdot\cot^2 35^\circ=-1$

6. $\overrightarrow{BC}=\overrightarrow{BA}+\overrightarrow{AC}=-\overrightarrow{AB}+\overrightarrow{AC}$
   $=-(-1,3)+(4,-9)=(5,-12)$
   $|\overrightarrow{BC}|=\sqrt{5^2+(-12)^2}=13$

7. L_1：$4x+3y+a=0 \Rightarrow L_1$：$8x+6y+2a=0$
   $d(L_1,L_2)=\frac{1}{2}=\frac{|2a-5|}{\sqrt{8^2+6^2}} \Rightarrow |2a-5|=5$
   $\Rightarrow 2a-5=\pm 5 \Rightarrow a=5$ 或 0(不合)
   L_1：$4x+3y+5=0$
   令 $y=0$ 代入 $L_1 \Rightarrow x=-\frac{5}{4}$ (x 截距)
   令 $x=0$ 代入 $L_1 \Rightarrow y=-\frac{5}{3}$ (y 截距)
   因此 L_1 與兩坐標軸所圍三角形面積
   $=\frac{1}{2}\cdot|-\frac{5}{4}|\cdot|-\frac{5}{3}|=\frac{25}{24}$

8. 利用分離係數法作除法如下：

$$\begin{array}{r|llll} & 1-1 & & & \\ 1-1+1 & 1-2 & +2a & & +b \\ & 1-1 & +1 & & \\ \hline & & -1 & +(2a-1) & +b \\ & & -1 & +1 & -1 \\ \hline & & & (2a-2) & +(b+1) \end{array}$$

   $\Rightarrow \begin{cases} 2a-2=-6 \\ b+1=3 \end{cases} \Rightarrow \begin{cases} a=-2 \\ b=2 \end{cases}$，$\therefore a+b=0$

9. $\begin{vmatrix} 6 & -5 & 1 \\ 4 & 8 & -4 \\ 4 & -1 & 3 \end{vmatrix}$ （第1列 $\times 4$ 加至第2列，$\times(-3)$ 加至第3列）$=\begin{vmatrix} 6 & -5 & 1 \\ 28 & -12 & 0 \\ -14 & 14 & 0 \end{vmatrix}$
   $=1\times\begin{vmatrix} 28 & -12 \\ -14 & 14 \end{vmatrix}-0\cdot\begin{vmatrix} 6 & -5 \\ -14 & 14 \end{vmatrix}+0\cdot\begin{vmatrix} 6 & -5 \\ 28 & -12 \end{vmatrix}$
   $=4\times 14\times\begin{vmatrix} 7 & -3 \\ -1 & 1 \end{vmatrix}=4\times 14\times 4=224$

10. $i^4=1$
    原式 $=|\frac{5i-4i+1}{8i-5i-3}|=|\frac{1+i}{3(-1+i)}|=\frac{|1+i|}{|3(-1+i)|}=\frac{\sqrt{2}}{3\sqrt{2}}=\frac{1}{3}$

11. $(2\sin\frac{2\pi}{3},2\cos\frac{2\pi}{3})=(\sqrt{3},-1)$
    $=(2\times\frac{\sqrt{3}}{2},2\times(-\frac{1}{2}))=(2\cos\frac{11}{6}\pi,2\sin\frac{11}{6}\pi)$
    $\therefore r=2$，$\theta=\frac{11}{6}\pi+2n\pi$，$n\in Z$
    故當 $n=-1$ 時，可得極坐標 $(2,-\frac{\pi}{6})$

12. $\frac{5x^2+3x}{(x+2)(x^2-x+1)}=\frac{A}{x+2}+\frac{Bx+C}{x^2-x+1}$
    $\Rightarrow 5x^2+3x=A(x^2-x+1)+(Bx+C)(x+2)$
    $=(A+B)x^2+(-A+2B+C)x+(A+2C)$
    $\Rightarrow \begin{cases} A+B=5 \\ -A+2B+C=3 \\ A+2C=0 \end{cases} \Rightarrow \begin{cases} A=2 \\ B=3 \\ C=-1 \end{cases}$
    $\therefore A+B+C=2+3+(-1)=4$

13. $\begin{vmatrix} 4 & 2 & a+3 \\ 1 & -2 & b \\ 0 & 1 & 3 \end{vmatrix} = \begin{vmatrix} 4 & 2 & a+3 \\ 1 & -2 & b+0 \\ 0 & 1 & 2+1 \end{vmatrix}$

$= \begin{vmatrix} 4 & 2 & a \\ 1 & -2 & b \\ 0 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 2 & 3 \\ 1 & -2 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 5+(-8)+3-2=-2$

14. $\because \alpha$ 為 $4x^2-(3m+i)x+m-i=0$ 的實根

$\Rightarrow 4\alpha^2-(3m+i)\alpha+m-i=0$

$\Rightarrow (4\alpha^2-3m\alpha+m)-i(\alpha+1)=0$

$\Rightarrow \begin{cases} 4\alpha^2-3m\alpha+m=0 \\ \alpha+1=0 \end{cases} \Rightarrow \alpha=-1$

15. $\frac{1}{5}<x<\frac{1}{3}$ 為 $(x-\frac{1}{5})(x-\frac{1}{3})<0$ 的解

$\therefore ax^2+8x+b>0 \Leftrightarrow (x-\frac{1}{5})(x-\frac{1}{3})<0$

$\Leftrightarrow (5x-1)(3x-1)<0 \Leftrightarrow 15x^2-8x+1<0$

$\Leftrightarrow -15x^2+8x-1>0 \Rightarrow a=-15$，$b=-1$

$\therefore a+b=-16$

16. $x>-1$，由算幾不等式得 $\dfrac{(x+1)+\frac{9}{x+1}}{2} \geq \sqrt{(x+1)\cdot\frac{9}{x+1}}$

$\Rightarrow (x+1)+\frac{9}{x+1} \geq 2\sqrt{(x+1)\cdot\frac{9}{x+1}}=6 \Rightarrow x+\frac{9}{x+1} \geq 5$

17. $\overline{PQ}=\sqrt{(\sin 65°-\cos 85°)^2+(\cos 65°-\sin 85°)^2}$

$=\sqrt{\sin^2 65°-2\sin 65°\cos 85°+\cos^2 85°+\cos^2 65°-2\cos 65°\sin 85°+\sin^2 85°}$

$=\sqrt{2-2(\sin 65°\cos 85°+\cos 65°\sin 85°)}$

$=\sqrt{2-2\sin 150°}=\sqrt{2-2\times\frac{1}{2}}=1$

18. $\omega=\frac{-1+\sqrt{3}i}{2} \Rightarrow \omega^3=1$ 且 $1+\omega+\omega^2=0$

$(3-\omega)(3-\omega^2)(3-\omega^4)(3-\omega^8)$

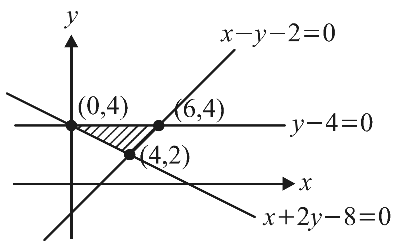
$=(3-\omega)(3-\omega^2)(3-\omega)(3-\omega^2)=[(3-\omega)(3-\omega^2)]^2$

$=(9-3\omega-3\omega^2+\omega^3)^2$

$=[10-3(\omega+\omega^2)]^2=[10-3(-1)]^2=169$

19. $5-2y \leq x-3 \leq y-1 \leq 3$

$\Leftrightarrow \begin{cases} x+2y-8 \geq 0 \\ x-y-2 \leq 0 \\ y-4 \leq 0 \end{cases}$ 所圍區域如下圖



$\therefore$面積為 $\frac{1}{2}\times 6\times 2=6$

20. $\overline{AC}=b=\sqrt{3}+1$，$\overline{BC}=a=\sqrt{2}$，$\angle C=45°$

由餘弦定理得

$c^2=(\sqrt{2})^2+(\sqrt{3}+1)^2-2\cdot\sqrt{2}\cdot(\sqrt{3}+1)\cos 45°$

$=2+4+2\sqrt{3}-2\sqrt{2}(\sqrt{3}+1)\cdot\frac{1}{\sqrt{2}}=4 \Rightarrow c=2$

又由正弦定理得

$\frac{\sqrt{2}}{\sin A}=\frac{2}{\sin 45°} \Rightarrow \sin A=\frac{\sqrt{2}\sin 45°}{2}=\frac{1}{2}$

21. $\sqrt{3-\sqrt{8}}=\sqrt{3-2\sqrt{2}}=\sqrt{(\sqrt{2}-1)^2}=\sqrt{2}-1$

$$\begin{array}{rrrrr|l}
1 & +5 & +6 & -1 & +1 & \\
 & -1 & -4 & -2 & +3 & -1 \\ \hline
1 & +4 & +2 & -3 & \boxed{+4} & \\
 & -1 & -3 & +1 & & \\ \hline
1 & +3 & -1 & \boxed{-2} & & \\
 & -1 & -2 & & & \\ \hline
1 & +2 & \boxed{-3} & & & \\
 & -1 & & & & \\ \hline
1 & \boxed{+1} & & & &
\end{array}$$

$\Rightarrow f(x)=(x+1)^4+(x+1)^3-3(x+1)^2-2(x+1)+4$

$\therefore f(\sqrt{3-\sqrt{8}})=f(\sqrt{2}-1)$

$=(\sqrt{2})^4+(\sqrt{2})^3-3(\sqrt{2})^2-2(\sqrt{2})+4$

$=4+2\sqrt{2}-6-2\sqrt{2}+4=2$

[另解]

令 $x=\sqrt{3-\sqrt{8}}=\sqrt{3-2\sqrt{2}}=\sqrt{(\sqrt{2}-1)^2}=\sqrt{2}-1$

$\Rightarrow x+1=\sqrt{2}$

$\Rightarrow (x+1)^2=(\sqrt{2})^2=2$

$\Rightarrow x^2+2x-1=0$

又 $f(x)=x^4+5x^3+6x^2-x+1$

$=(x^2+2x-1)(x^2+3x+1)+2$

$\therefore f(\sqrt{3-\sqrt{8}})=f(\sqrt{2}-1)=2$

22. 令 $\Delta=\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$，$\Delta_x=\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$

$\Delta_y=\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$，$\Delta_z=\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$

$\Rightarrow \frac{\Delta_x}{\Delta}=2$，$\frac{\Delta_y}{\Delta}=1$，$\frac{\Delta_z}{\Delta}=4$

方程組 $\begin{cases} 2a_1x+b_1y+4c_1z=3d_1 \\ 2a_2x+b_2y+4c_2z=3d_2 \\ 2a_3x+b_3y+4c_3z=3d_3 \end{cases}$ 之解為

$$x=\frac{\begin{vmatrix} 3d_1 & b_1 & 4c_1 \\ 3d_2 & b_2 & 4c_2 \\ 3d_3 & b_3 & 4c_3 \end{vmatrix}}{\begin{vmatrix} 2a_1 & b_1 & 4c_1 \\ 2a_2 & b_2 & 4c_2 \\ 2a_3 & b_3 & 4c_3 \end{vmatrix}}=\frac{12}{8}\cdot\frac{\Delta_x}{\Delta}=\frac{12}{8}\cdot 2=3$$

$$y=\frac{\begin{vmatrix}2a_1 & 3d_1 & 4c_1\\ 2a_2 & 3d_2 & 4c_2\\ 2a_3 & 3d_3 & 4c_3\end{vmatrix}}{\begin{vmatrix}2a_1 & b_1 & 4c_1\\ 2a_2 & b_2 & 4c_2\\ 2a_3 & b_3 & 4c_3\end{vmatrix}}=\frac{24}{8}\cdot\frac{\Delta_y}{\Delta}=3\cdot 1=3$$

$$z=\frac{\begin{vmatrix}2a_1 & b_1 & 3d_1\\ 2a_2 & b_2 & 3d_2\\ 2a_3 & b_3 & 3d_3\end{vmatrix}}{\begin{vmatrix}2a_1 & b_1 & 4c_1\\ 2a_2 & b_2 & 4c_2\\ 2a_3 & b_3 & 4c_3\end{vmatrix}}=\frac{6}{8}\cdot\frac{\Delta_z}{\Delta}=\frac{6}{8}\cdot 4=3$$

$\therefore a=3$，$b=3$，$c=3$

23. 由柯西不等式知：

$[a^2+b^2+(3-a-b)^2](1^2+1^2+1^2)\geq[a+b+(3-a-b)]^2$

$\Rightarrow a^2+b^2+(3-a-b)^2\geq\frac{9}{3}=3$

24. $|\vec{u}|=1$，$|\vec{v}|=1$，設$\vec{u}$、$\vec{v}$之夾角為$\theta(0\leq\theta\leq\pi)$

$\vec{u}\cdot\vec{v}=|\vec{u}||\vec{v}|\cos\theta=\cos\theta$

$\because -1\leq\cos\theta\leq 1$，$\therefore -1\leq\vec{u}\cdot\vec{v}\leq 1$

$|\vec{u}-\vec{v}|^2=|\vec{u}|^2+|\vec{v}|^2-2\vec{u}\cdot\vec{v}=2-2\vec{u}\cdot\vec{v}$

$\Rightarrow 0\leq|\vec{u}-\vec{v}|^2\leq 4\Rightarrow 0\leq|\vec{u}-\vec{v}|\leq 2$

故$|\vec{u}-\vec{v}|$之最大值為 2

25. $x^3+1=(x+1)(x^2-x+1)$

設 $f(x)=x^{100}+x^{50}+2=(x^3+1)\cdot Q'(x)+mx^2+nx+r$

令 $x^3=-1$ 代入上式

$\Rightarrow mx^2+nx+r=(-1)^{33}\cdot x+(-1)^{16}\cdot x^2+2=x^2-x+2$

$f(x)=(x^3+1)\cdot Q'(x)+x^2-x+2$

$=(x+1)(x^2-x+1)\cdot Q'(x)+x^2-x+2$

$=(x+2)(x^2-x+1)\cdot Q'(x)+1$

$=(x^2-x+1)\cdot Q(x)+ax+b$

$\Rightarrow a=0$，$b=1$，$\therefore 2a+b=1$